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Application of Response Surface Methodology in Optimization of the Yields of Common Bean (*Phaseolus vulgaris L.)* Using Animal Manures

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Abstract The objective of design and analysis of experiments is to optimize a response variable which is influenced by several independent variables. In agriculture, many statistical studies have focused on investigating the effect of application of organic manure on the yield and yield components of crops. However, many of these studies do not try to optimize the application of the manures for maximum productivity, but select the best treatment among the treatment range used. This is mainly due to design and analysis of experiments applied. Therefore, there is a need to apply a statistical method that would establish the effect of the application of organic manures on crop production and in addition optimize the levels of application of these manures for maximum productivity. This study aimed at application of response surface methodology for optimization of the yields of common bean (*Phaseolus vulgaris* L.) using animal manure. The study was conducted at Chuka University Horticultural Demonstration Farm. The experiment was laid down in a Randomized Complete Block Design. The treatments consisted of three organic manure sources (cattle manure, poultry manure and goat manure) each at three levels (0, 3 and 6 tonnes per ha). Data was collected from six weeks after sowing to physiological maturity. Data was collected on the weight of the grain yield harvested in each experimental plot measured by use of a weighing scale. The data collected was analysis using the R-statistical software. The study findings indicated that animal manures had a significant effect ($p < 0.05$) on the yield of common beans. The results also showed that the optimum levels of application of the manures in the area of study were 2.1608 t ha^{-1}, 12.7213 t ha^{-1} and 4.1417 t ha^{-1} cattle manure, poultry manure and goat manure, respectively. These were the optimum levels that would lead to maximum yield of common beans without an extra cost of input.

Keywords: *organic manure, yield, yield components, response surface methodology, common bean, randomized complete block design*

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1. Introduction

Statistical methods have been used to examine the relationship between application of organic manure and crop productivity [10], [2], [3] & [16]. However, most of these studies have focused on the effect of application of organic manure on the yield and yield components of various crops. For instance, split plot designs and randomized complete block designs had been applied to investigate the effect of organic and inorganic fertilizers on the yield and yield components of maize [10]. This study did not attempt to optimize levels of farm yard manure or the inorganic fertilizers using the treatment range applied for maximum production of maize. However, the objective of design and analysis of experiments is to optimize a response (output variable) which is affected by numerous independent variables (input variables).

[2] also applied randomized complete block design to investigate the effect of different levels of sulphur, nitrogen and compost manure on the yield and yield components of maize. Just like in [10], the positive effect was established but there was no optimization on the levels of fertilizer and compost manure that would lead to the highest response. The effect of chicken manure, NPK fertilizers and their combinations on common beans, using a randomized complete block design, demonstrated variable bean crop performance [3]. However, due to limitation of design and analysis of experiments used in this study, the study only established the best treatments among the treatment range applied. However, no optimization was done in this study and thus the levels

of the manures that would yield the best production remained unknown.

Optimization of levels of manures and fertilizers is important since it enables the farmers to get the best production without an extra cost in input. Response surface methodology is applied in optimization of response surfaces in a situation where y is the response variable of interest and there is a set of predictor variables $x_1, x_2 \ . \ . \ . \ x_k$ [15]. However, in several Response Surface Methodology studies, the form of the relationship between the response and the independent variables is unknown [13]. The main objective of Response Surface Methodology is to find the operating conditions for the system that are optimum or to find a section of the space factor in which operating requirements are satisfied [13]. The first-order model is expected to be suitable when the experimenter is concerned in approximating the response surface that is true over a moderately small section of the variable space of independent variable in a setting where there is almost no curvature in the response function [13].

In most of the cases, the curvature in the true response surface is strong enough that the first-order model (even with the interaction term included) is inadequate [15]. A second-order model will likely be required in these situations. Such a response surface could arise in approximating a response such as yield, where we would expect to be operating near a maximum point on the surface [11]. Thus, this study applied RSM to establish the best levels of the manures that optimized the yields of common beans.

2. Methodology

The study used Randomized Complete Block Design (RCBD), with 27 treatments (cow manure, poultry manure and goat manure) each at three levels (0, 3 and 6 tonnes per hectare), and replicated three times. This study also used KATX 56 bean variety from Kenya Agricultural and Livestock Research Organisation (KALRO) in Embu. The weight of the grain yield harvested in each experimental plot was measured by use of the weighing scale. The collected data was subjected to analysis using R software. The analysis involved fitting second order models that were later used to obtain the optimal settings of the input variables that would maximize the yield of common beans. The visualization of this optimization was done by the use of response surface contour plots.

The equations for the fitted second order polynomial models are represented as;

$$\begin{aligned} Y_i = {} & \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 \\ & + \alpha_{11} X_1^2 + \alpha_{22} X_2^2 + \alpha_{33} X_3^2 \\ & + \alpha_1\alpha_2 X_1 X_2 + \alpha_1\alpha_3 X_1 X_3 + \alpha_2\alpha_3 X_2 X_3 + \varepsilon_i \end{aligned} \quad (1)$$

where Y_i = Response values , X_1 = Cattle manure; X_2 = Goat manure; X_3 = Poultry manure; β_0, a_0 = constant value; α_i = linear coefficients, α_{ii} = Quadratic coefficients $\alpha_i\alpha_j$ = Cross product ($i < j$) and ε_i = error terms.

The parameters of the model 1 are estimated by means of least squares method. An empirical model of second order can be presented as;

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \ldots + \beta_k x_{ik} + \varepsilon_i. \quad (2)$$

This model can also be represented as;

$$\beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i. \quad (3)$$

From equation 3, the error sum of squares can be written as;

$$L = \sum_{i=1}^{k} \varepsilon_i^2 \quad (4)$$

$$= \sum_{i=1}^{k} \left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij} \right)^2 \quad (5)$$

The parameters β' s are estimated by least squares method after minimising the error sum of squares presented in equation 5.

The adequacy and goodness of fit of the fitted models will be done using R^2, R^2_{adj} and Lack of Fit test statistic presented in equations 6, 7 and 8.

$$R^2 = 1 - \frac{SS_{residual}}{SS_{model} + SS_{residual}} \quad (6)$$

$$R^2_{adj} = 1 - \frac{n-1}{n-p}\left(1 - R^2\right) \quad (7)$$

$$\begin{aligned} F_{computed} &= \frac{Lack\ of\ fit\ sum\ of\ squares / (m-p)}{Pure\ error\ sum\ of\ squares / (n-m)} \\ &= \frac{Mean\ sum\ of\ squares\ for\ lack\ of\ fit}{Mean\ sum\ of\ squares\ for\ pure\ error}. \end{aligned} \quad (8)$$

Where SS is the sum of the squares, n the number of experiments, and p the number of predictors (term) in the model.

Optimization of the response in the second order model is done by obtaining a mathematical solution of the stationary points. This is done by first expressing the second order model in matrix notation as shown in equation 9.

$$\hat{y} = \hat{\beta}_0 + X'b + X'BX \quad (9)$$

Where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}, b = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}, B = \begin{bmatrix} \hat{\beta}_{11}, \hat{\beta}_{11}/2, & \cdots & , \hat{\beta}_{1k}/2 \\ \vdots & \ddots & \vdots \\ sym & \cdots & \hat{\beta}_{kk} \end{bmatrix}.$$

The derivative of $\hat{y}$ with respect to the elements of the vector X and then equated to 0 is represented in equation 10.

$$\frac{\partial \hat{y}}{\partial x} = b + 2Bx = 0. \quad (10)$$

The stationary points are obtained by getting the solution to equation 10. This solution is presented in equation 11.

$$x_s = -\frac{1}{2}B^{-1}b. \quad (11)$$

The obtained stationary points are the optimal settings of the input variables that would lead to maximum value of the output variables. Obtaining of the maximum response using the stationary values is presented in equation 12.

$$\hat{y}_s = \hat{\beta}_0 + \frac{1}{2}x_s^{'}b. \quad (12)$$

3. Results and Discussion

3.1. Preliminary Analysis of the Yield of Common Beans

The data collected was analyzed for average yield grouped by factors (Table 1). The aim was to find out which factor yielded the highest mean The average yield grouped by factors for cattle manure at level 0 was 13.11 tonnes per hectare, at level 3 the average yield was 13.43 and at level 6 the average yield was 13.99 tonnes per hectare (Table 1). For poultry, the average yield at level 0 was 12.45 tonnes per hectare, at level 3, it was 13.56 tonnes per hectare and 18.51 tonnes per hectare at level 6 (Table 1). For goat manure at level 0, yields were 11.80 tonnes per hectare, at level 3 it was 13.43 tonnes per hectare and at level 6, the yield was 15.30 tonnes per hectare (Table 1). The skewness ranged from 0.6443 to 1.2472 (Table 1). Kurtosis values ranged from -0.6542 to 1.1717 (Table 1).

The results showed that poultry manure and goat manure had a positive effect on bean grain yield (Table 1). Regarding the normality test, the results indicated that all the variables were normally distributed since their skewness test values lied within the range of ±3. Also, the Kurtosis test values were within the threshold range of either ±1 and, or ±2 (Table 1).

The significance of the factors was tested using the analysis of variance at 5% level of significance (Table 2). The least significant differences test was used to separate the factor means to find out which factor had a better effect on the yield of common beans (Table 3).

The analysis of variance showed that there is significant ($p < 0.05$) effect of factors levels on the yield of common beans (Table 2). The least significant differences showed that the mean yields for common beans were different for the various manure levels.

Table 1. Effects of organic manure factor levels on grain yield (weight in tonnes per hectare)

Factor	Mean	Standard Deviation	Median	Skewness	Kurtosis	Maximum	Minimum
C0	13.11	9.74	8.60	1.2472	1.1717	26.9	1.5
C3	13.43	9.84	10.30	1.541	1.7147	30.8	2.3
C6	13.99	10.20	12.20	1.0731	0.45	40.5	1.8
P0	12.45	5.28	6.55	1.5731	1.4614	28.5	1.9
P3	13.56	9.13	10.85	1.3005	1.7085	41.7	3.3
P6	18.51	11.55	15.75	0.6443	-0.6542	46.9	2.8
G0	11.80	9.14	8.70	1.7405	1.0542	28.6	1.5
G3	13.43	10.36	9.30	1.2217	0.7514	36.8	1.8
G6	15.29	9.96	12.65	1.0218	0.3351	42.5	3.1

$Ci(i = 0, 3, 6)$ = Cattle manure levels, $Gi(i = 0, 3, 6)$ = Goat manure levels, $Pi(i = 0, 3, 6)$ = Poultry manure levels.

Table 2. ANOVA for the factor effect on yield of common beans

	df	SS	MS	F value	p-value
Block	1	345.9712	345.9712	6.809376	0.103543
Factors levels	26	8606.941	331.0362	6.515426	1.16E-12
Residuals	108	5487.271	50.80806		

Table 3. Means of common bean yield (weight in tonnes per hectare) across different factors levels

Factor	**Factor Levels**	**Mean**
	0	13.11a
Cattle	3	13.43a
	6	13.99b
	0	12.45c
Poultry	3	13.56d
	6	18.51e
	0	11.80f
Goat	3	13.43g
	6	15.29h

[a]Means followed by same letter are not significantly different from each other at 5% probability level.

3.2. Model Fitting for the Yield of Common Beans

The coefficients for the fitted second order model for cattle, poultry, goat, their interaction and the quadratic of cattle, poultry and goat manures on grain yields and their corresponding p-values were are presented in Table 4.

Table 4. Second Order Response Surface Methodology Model for Grain Yield (weight in tonnes per hectare)

	Estimate	Std Error	t- value	p-value
Intercept	13.4025	1.7688	7.5773	<0.001
Cattle	0.4417	0.8188	0.5394	0.59039
Poultry	5.033	0.8188	6.1473	<0.001
Goat	1.7463	0.8188	2.1328	0.0346
Cattle:poultry	1.45	1.0028	1.4459	<0.001
Cattle:goat	4.1444	1.0028	4.1329	0.1503
Poultry: goat	1.7458	1.0028	1.7410	0.03871
Cattle^2	0.1231	1.4182	0.0868	0.9309
Poultry ^2	-0.0852	1.4182	-0.0601	0.9522
Goat^2	0.1204	1.4182	0.0849	0.9325

The model can be presented as;

$$\begin{aligned} Y_1 &= 13.40 + 0.4412X_1 + 5.033X_2 + 1.7463X_3 \\ &+ 1.45X_1X_2 + 4.1444X_1X_3 + 1.7458X_2X_3 \\ &+ 0.1231X_1^2 - 0.0852X_2^2 + 0.1204X_3^2 \end{aligned}$$

Where, Y_1 = weight of the grain yield; X_1 = cattle manure; X_2 = poultry manure; X_3 = goat manure.

The regression coefficient estimates show that for a unit change in cattle manure, poultry manure and goat manure, grain yield of common beans would increase by unit factors of 0.411, 5.033 and 1.7463 respectively. This implies that poultry manure is slightly more effective than goat manure. In addition, it was found that combined application of poultry and goat manure had a regression coefficient value of 1.7458 and a p-value of 0.03871 < 0.05, hence statistically significant at 5% significance level. This implies that for one unit change in combined poultry and goat manure, grain yield for common beans would increase by a factor of 1.7458. Similarly, combined application of poultry and cattle manure had a regression coefficient value 4.144 and a p-value of < 0.0001 < 0.05, hence statistically significant at 5% significance level. This implies that for one unit change in combined application of cattle and poultry manure, grain yield of common beans would increase by a factor of 4.144. This shows that combined poultry and cattle manure is much more effective than combined poultry and goat manure and also cattle and goat manure. However, it was observed that the quadratic terms were not significant.

Since the quadratic terms were not significant, so a first order with interaction terms only was performed. Since the two interaction terms were significant, it was necessary to fit a first order response surface model with two-way interactions.

The findings showed that poultry manure (p < 0.001), goat manure (p = 0.0328) and the interaction between cattle and poultry (p =0.0407), interaction between poultry and goat (p < 0.001) had a significant effect on yield (Table 5).

Table 5. First Order with Two-Way Interactions Response Surface Methodology Model for the Grain Yield (Weight)

	Estimate	Std Error	t-value	p-value
Intercept	13.508	0.6621	20.4026	<0.001
Cattle	0.4417	0.8109	0.5447	0.5868
Poultry	5.0333	0.8109	6.2073	<0.001
Goat	1.7463	0.8109	2.153	0.0328
Cattle: poultry	1.45	0.9931	1.4601	0.0407
Cattle: goat	4.1444	0.9931	4.173	0.1463
Poultry: goat	1.7458	0.9931	1.7579	<0.001

3.3. Model Diagnostics for the Fitted Model

First order with two-way interactions ANOVA table for grain yield was also done and the results showed that the F statistics values for first order RSM and two-way interaction were 11.384 and 5.93 respectively with corresponding p-values as <0.001 (Table 6). The F statistic and p-values for the lack of fit were 0.8855 and 0.6055 respectively (Table 6).

Table 6. Analysis of Variance of First Order with Two-Way Interactions Response Surface Methodology Model for Grain Yield (Weight)

	df	SS	MS	F value	p-value
FO (x_1, x_2, x_3)	3	3086.5	1028.85	11.384	<0.001
TWI(x_1, x_2, x_3)	3	1607.5	535.84	5.93	<0.001
Residuals	155	14007.6	90.372		
Lack of fit	20	912.9	45.65	0.8855	0.6055
Pure Error	135	7093.9	52.54		

FO = First order, TWI = Two way interactions.

The ANOVA test results showed that the lack of fit test was insignificant with a p-value = 0.6055. The first order and two-way interaction (p-value<0.001) were significant at 5% level of significance with their p-values less than 0.05. Therefore the study found that there is no significant lack of fit in the model and so the study concluded that the reduced model was adequate since it satisfied the adequacy conditions in non-linear form.

3.4. Optimal Application Different Levels of Organic Manure that Maximizes Yields for Common Beans

The optimal settings of the input variables that would lead to maximum output were obtained in form of stationary points from the fitted model. The visualization of the optimal settings was done in form a contour plot (Figure 1).

The findings of this study showed that poultry manure and goat manure have a direct effect on the grain yield to a certain level and then with more increase, led to decrease in poultry manure and goat manure. The optimal rate of application of cattle manure, poultry manure, and goat manure was 2.1608, 12.7213 and 4.1417 tonnes per hectare respectively. These findings agree with findings by [6] and Makinde [12] in that organic manures have a direct effect on crop yields.

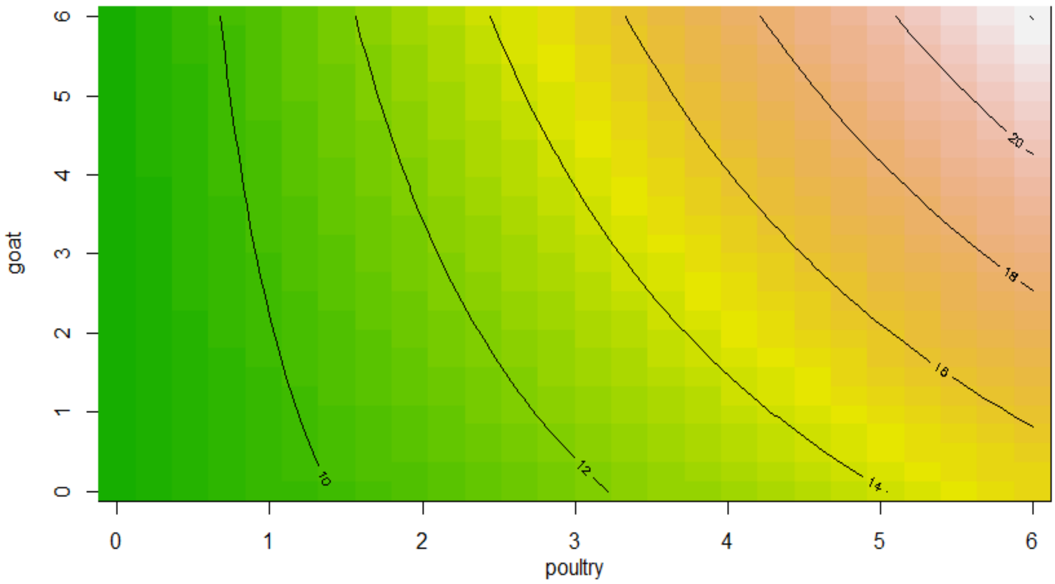

Figure 1. Response surface contour plot for grain yield as a function of goat manure and poultry manure at constant level of cattle manure

4. Conclusion

In conclusion, response surface methodology (RSM) is a relevant tool as far as the optimizations of the input variables that maximizes the output variable is concerned. In this study RSM was successfully used in optimization of the yields of common bean using animal manures. The optimal conditions for the production of common beans were obtained as stationary points from the fitted model and visualized graphically on a contour response surface plot. From the study, it was also concluded that goat and poultry manure had a positive impact on the yield and the yield components of common beans.

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