CHUKA

UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE BACHELOR OF

MATH 420: PARTIAL DIFFERENTAL EQUATION 1

STREAMS: **TIME: 2 HOURS**

DAY/DATE:

INSTRUCTION: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTION

QUESTION ONE   30MARKS (COMPULSORY)

a) Show that the parametric equation $x = a\, sinu\, cosv, \quad y = a\, sinu\, cosv,$ $z = a cosu$ represent a spherical surface   (4marks)

b) Find the direction cosines normal to the surface $x^2 + y^2 + z = a^2$ at the point $(1,1,1,)$
(5marks)

c) Solve the $\frac{dx}{6(y-z)} = \frac{2dy}{3(z-x)} = \frac{3dz}{2(x-y)}$ by finding two integral surfaces c_1 and c_2 (6marks)

d) Solve the Pfaffian differential equation $2x(x^3 + y^3)dx + (3x^2y^2 + 3y^4)dy = 0$
(4marks)

e) Form a first order differential equation by eliminating a constant a of $z = a(x + y)^2 + b$
(5marks)

f) Show that the equation $xp = yq, \quad z(xp + yq) = 2xy$ are compatible   (6marks)

QUESTION TWO [20MARKS]

a) Determine the orthogonal trajectory of the line of intersection of the cone $x^2 + y^2 + z = a$ with the family of planes parallel to the xy plane ie the plane $z = 0$ (8marks)

b) Find the complete integral of the equation $p^2x + q^2y = z$ using Charpit's method
(7marks)

c) Find the integral curve of the equation $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$ (5marks)

QUESTION THREE [20MARKS]

a) Show that the direction cosines of the tangents of the curve of intersection of $x + y + z = 1$ and the conic $ax^2 + by^2 + cz^2 = 1$ are proportional to $(by - cz, \ cz - ax, \ ax - by)$ (7marks)

b) Find the general solution of the semilinear equation $x^2 p + y^2 q = (x + y)z$ (8marks)

c) Solve the Pfaffian differential equation   (5marks)

QUESTION FOUR [20MARKS]

a) Find the surface which intersect with the surface of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, \quad z = 1$ (10marks)

b) Show the equation $(yz + z^2)dx - xzdy + xydz = 0$ is homogenous   (4marks)

c) Determine the characteristic equations of $z = p^2 - q^2$ (6marks)